

# Model-based Identification, Estimation, and Control for Large-scale Urban Road Networks

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**Abstract**—Network-level road traffic control remains a challenging problem. Macroscopic fundamental diagram (MFD) based dynamical models of large-scale urban networks enable development of model predictive perimeter control methods, which represent an efficient congestion control solution with substantial potential for practical implementation. In this paper we propose a model-based system identification method for computing the MFD parameters given measurements on historical trajectories of the traffic state and inflow demand. Furthermore, nonlinear moving horizon estimation (MHE) and model predictive control (MPC) formulations for MFD-based dynamics are presented, which enable high-performance traffic control under severe measurement noise. Microsimulation-based case studies, considering an urban network with 1500 links, where the MFD parameters obtained by the identification method are used in MHE and MPC design, demonstrate the operation of the proposed framework.

## I. INTRODUCTION

Modeling and control of road traffic in large-scale urban networks present considerable challenges. Large network size, inadequate infrastructure and coordination, spatiotemporal propagation of congestion, and the interaction between driver decisions and the traffic control system contribute to the difficulties faced when creating realistic models and designing effective control schemes for urban networks. Although considerable research has been directed towards designing efficient real-time traffic control schemes in the last decades (see [1] for a review), dynamical modeling and control design for heterogeneously congested networks at the city level remains a challenging problem.

Substantial research effort has been directed towards developing methods for modeling and control of urban traffic, which usually focus on mesoscopic models keeping track of link-level traffic dynamics with control strategies using local information. Based on the linear-quadratic regulator (LQR) problem, traffic-responsive urban control [2] represents a multivariable feedback regulator approach for network-wide urban traffic control, which has been tested both via simulations and field implementations (see [3], [4]). Inspired by the max pressure routing scheme for wireless networks [5], many local traffic control schemes have been proposed for networks of signalized intersections (see [6], [7]), which involve evaluations at each intersection requiring information

exclusively from adjacent links. Although the high level of detail in mesoscopic traffic models is desirable for simulation purposes, the increased model complexity results in complications for control, whereas local control strategies might not be able to operate properly under heavily congested conditions, as they do not protect the congested regions upstream. Another disadvantage of sophisticated local controllers is the need for highly detailed information on traffic states of the links, which are difficult to estimate or measure.

Considering severe model uncertainty, difficulty of instrumentation, and excessive computational burden associated with detailed link-level modeling and control methods that need to consider all intersections and traffic lights for managing traffic in the entire city, such traffic management approaches appear to be practically infeasible. As an alternative to these link-level approaches, network-level methods employing perimeter control (i.e., control with actuation over a set of traffic lights on the perimeter between two neighborhood-sized areas) are receiving increasing attention as practicable approaches for city-wide traffic control. Based on macroscopic modeling of heterogeneously congested urban road networks, perimeter control involves manipulation of macroscopic traffic flows (i.e., rate of vehicles transferring between neighborhood-sized areas). However, employing aggregated modeling and control approaches using only a small subset of all intersections as actuators, the perimeter control method shows substantial promise in alleviating congestion and improving mobility in large-scale urban networks.

In the first step of the method, a heterogeneously congested city-sized road network is partitioned into a set of regions with homogeneous distribution of congestion, enabling development of macroscopic traffic models. Then, a set of traffic lights on the boundaries (i.e., at the perimeters of the regions) between the regions are instrumented to be used as the actuators that can manipulate vehicle flows between the regions. After installing sensors over the city to measure the number of vehicles in each region (possibly also their destination regions), it is possible to construct feedback perimeter control systems for managing traffic and improving mobility at the city scale. Using various control design techniques, many perimeter control (or gating) methods have been developed for single-region [8], [9], [10] and multi-region [11], [12], [13] urban networks.

Macroscopic fundamental diagram (MFD) of urban traffic emerged as the primary modeling tool enabling development of aggregated modeling and control approaches for large-scale traffic dynamics. An urban region with roughly homogeneous accumulation (i.e., small spatial link density het-

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erogeneity) can be modeled using the MFD, which provides a unimodal, low-scatter, and demand-insensitive relationship between accumulation and trip completion flow. MFD as a concept was first proposed in [14], and experimentally proven to exist for urban areas recently in [15]. Using MFD it is possible to express the rate of vehicles exiting traffic in a region (either through ending the trip inside the region or transferring to an adjacent region) as a function of the region accumulation [15].

Although a powerful modeling tool, the MFD has also its challenges that might undermine its accuracy in expressing urban traffic dynamics: (a) Heterogeneous distribution of accumulation, especially in congested conditions, leads to the loss of a well-defined MFD for the urban region (see [16], [17], [18], [19]), (b) hysteresis phenomena leading to different behaviors in the MFD shape for the onset and offset of congestion (see [20], [21]). While heterogeneously congested networks do not exhibit well-defined MFDs, using clustering methods the network can be partitioned into a set of homogeneously congested (i.e., with low link density variance) regions, which can result in a set of well-defined MFDs for each region (see [22], [23], [24]). Despite its shortcomings, the MFD substantially reduces the complexity of traffic models by avoiding the need for considering the densities of individual links of the network, which number in the thousands for city-scale systems. Appearing thus as an efficient modeling tool for expressing aggregated dynamics of urban traffic, the MFD enables the design of model-based control methods for network-level road traffic management.

Application of model predictive control (MPC) to traffic control problems saw increased interest in the ITS literature in the last 15 years. Many MPC based methods for various settings of traffic control have been proposed: Ramp metering for freeway networks, variable speed limits, integrated route guidance and variable speed limits for freeway networks, and signal control for urban networks [25], [26], [27], [28], [29].

MFD-based MPC schemes for urban traffic began to appear recently in the literature: Nonlinear MPC for a two-region network actuated with perimeter control [13], hybrid MPC with perimeter control and switching signal timing plans [30], dynamical modeling of heterogeneity and hierarchical control with MPC on the upper level [31], MPC with MFD-based travel time and delays as performance measures [32], two-level hierarchical MPC with MFD-based and link-level models [33], multimodal MFDs network model-based MPC of city-scale ride-sourcing systems [34], MPC with perimeter control and regional route guidance [35] and extensions with a path assignment mechanism [36]. A more detailed literature review of MFD-based modeling and control can be found in [37].

Works on MFD-based control either employ control design based on known MFDs tested with macroscopic (i.e., MFD-based) simulations using artificial uncertainty on the MFDs ([13], [30], [38]), or control design based on MFDs obtained by fitting polynomials to historical data tested with microscopic simulations ([12], [39], [33], [40]). Thus, we identify two points as gaps to be addressed in the MFD-

based control literature: (1) Using system identification to estimate MFD-based model parameters remains unexplored, (2) although there is recent work considering state estimation with MFDs (see [38]), there are no results on state estimation with realistic tests employing microscopic simulation. Here we address the first point by a model-based parameter estimation (MBPE) formulation with MFD-based models, which employs the least squares prediction error method [41] from system identification literature. As an application example, the formulations of nonlinear moving horizon estimation (MHE) and economic nonlinear MPC schemes are given, which are intended to operate using the MFD parameters computed by the proposed MBPE method. Finally, realistic simulation studies, addressing the second point, employing the microscopic simulation package Aimsun are presented to demonstrate performance of the proposed methods.

## II. MODELING

Consider a city-scale road traffic network, consisting possibly of hundreds of links and intersections, with heterogeneous distribution of accumulation (i.e., number of vehicles) on its links. Empirical results indicate that the MFD can be approximated by an asymmetric unimodal curve skewed to the right [15], which can, for example, be chosen as a third degree polynomial:

$$g_i(n_i(t)) = a_i n_i^3(t) + b_i n_i^2(t) + c_i n_i(t), \quad (1)$$

where  $n_i(t)$  (vehicles; abbreviated henceforth as veh) is the accumulation of region  $i$ ,  $g_i(n_i(t))$  (veh/s) is the trip completion flow of the region (i.e., rate of vehicles exiting traffic), whereas  $a_i$ ,  $b_i$ , and  $c_i$  are model parameters.

Given a network  $\mathcal{R}$  consisting of a set of  $R$  regions ( $\mathcal{R} = \{1, 2, \dots, R\}$ ), each with a well-defined MFD, aggregated dynamical models of large-scale road traffic networks can be developed based on interregional traffic flows as the following vehicle conservation equations [13], [31] :

$$\dot{n}_{ii}(t) = q_{ii}(t) - m_{ii}(t) + \sum_{h \in \mathcal{N}_i} u_{hi}(t) m_{hii}(t) \quad (2a)$$

$$\begin{aligned} \dot{n}_{ij}(t) = & q_{ij}(t) - \sum_{h \in \mathcal{N}_i} u_{ih}(t) m_{ihj}(t) \\ & + \sum_{h \in \mathcal{N}_i; h \neq j} u_{hi}(t) m_{hij}(t), \end{aligned} \quad (2b)$$

where  $n_{ii}(t)$  (veh) and  $n_{ij}(t)$  (veh) are state variables expressing the accumulation in region  $i$  with destination region  $i$  and  $j$ , respectively (with  $n_i(t) = \sum_{j=1}^R n_{ij}(t)$ ),  $q_{ii}(t)$  (veh/s) and  $q_{ij}(t)$  (veh/s) are disturbances expressing the rate of vehicles appearing in region  $i$  demanding trips to destination region  $i$  and  $j$ , respectively,  $u_{ih}(t) \in [\underline{u}, \bar{u}]$  (with  $0 \leq \underline{u} < \bar{u} < 1$ ) are control inputs between each pair of adjacent regions  $i$  and  $h$  expressing actions of perimeter control actuators (with  $h \in \mathcal{N}_i$ ; where  $\mathcal{N}_i$  is the set of regions adjacent to  $i$ ) that can manipulate vehicle flows transferring between the regions,  $m_{ihj}(t)$  (veh/s) is the vehicle flow attempting to transfer from  $i$  to  $h$  with destination  $j$ :

$$m_{ihj}(t) \triangleq \theta_{ihj}(t) \frac{n_{ij}(t)}{n_i(t)} g_i(n_i(t)), \quad (3)$$

where  $\theta_{ihj}(t) \in [0, 1]$  is the route choice term expressing, for the vehicles exiting region  $i$  with destination  $j$ , the ratio that is transferring to region  $h$  (with  $m_{hii}(t)$  and  $m_{hij}(t)$  defined similarly), whereas  $m_{ii}(t)$  (veh/s) is the exit (i.e., internal trip completion) flow of region  $i$ :

$$m_{ii}(t) \triangleq \frac{n_{ii}(t)}{n_i(t)} g_i(n_i(t)). \quad (4)$$

Route choice effect can be omitted in modeling if the network topology leads to a single obvious route choice, in which case  $\theta_{ihj}(t) = 1$  for all time for only one region  $h \in \mathcal{N}_i$  for each  $i$ - $j$  pair (with  $j \neq i$ ). The focus in this paper is on those networks where route choice can be omitted (see [35] for a study where it is included).

Assuming additive process and measurement noise, the dynamics (2) and measurement can be written as:

$$\dot{n}(t) = f(n(t), q(t), u(t), p) + w(t), \quad (5)$$

$$y_n(t) = n(t) + v_n(t) \quad (6)$$

$$y_q(t) = q(t) + v_q(t) \quad (7)$$

where  $n \in \mathbb{R}^{R^2}$  (state) and  $q \in \mathbb{R}^{R^2}$  (measured disturbance) are the vectors of accumulations and inflow demands, respectively,  $u \in \mathbb{R}^{2 \cdot m_a}$  (control input) is the vector of transferring flow restrictions between adjacent regions via perimeter control actuators (with  $m_a$  the number of adjacent region pairs),  $p \in \mathbb{R}^{R \cdot m_p}$  is the model parameters vector (containing the MFD parameters  $a_i$ ,  $b_i$ , and  $c_i$  given in (1) for each region; with  $m_p$  the number of parameters associated with the MFD of one region),  $w \in \mathbb{R}^{R^2}$  is the process noise expressing uncertainty in the dynamics (with  $w \sim \mathcal{N}(0, \Sigma_w)$ ),  $y_n \in \mathbb{R}^{R^2}$  and  $y_q \in \mathbb{R}^{R^2}$  (measurement) are the measured values of  $n$  and  $q$ , respectively, whereas  $v_n \in \mathbb{R}^{R^2}$  and  $v_q \in \mathbb{R}^{R^2}$  are the measurement noise vectors (with  $v_n \sim \mathcal{N}(0, \Sigma_{v_n})$  and  $v_q \sim \mathcal{N}(0, \Sigma_{v_q})$ ).

### III. IDENTIFICATION, ESTIMATION, AND CONTROL

In this section we present formulations of model-based identification, estimation, and control for large-scale urban road networks with MFD-based dynamical models. Via identification the model parameters can be extracted from offline data, which can then in practice be used online for traffic management employing the presented estimation (for filtering out noise from real-time traffic state measurements) and control (for improving mobility by manipulating macroscopic traffic flows via perimeter control) methods.

#### A. System Identification

Based on the prediction error method [41], we can formulate the problem of obtaining the MFD parameters with the best least squares fit between the measured and predicted

trajectories of  $n(t)$  and  $q(t)$  as the following MBPE problem:

$$\begin{aligned} \min_{p, n_k, q_k} \quad & \sum_{k=0}^{K-1} \|w_k\|_Q^2 + \sum_{k=0}^K \left( \|v_{n,k}\|_{R_n}^2 + \|v_{q,k}\|_{R_q}^2 \right) \quad (8) \\ \text{s.t.} \quad & \text{for } k = 0, \dots, K : \\ & y_n(kT) = n_k + v_{n,k} \\ & y_q(kT) = q_k + v_{q,k} \\ & 0 \leq n_k \leq \bar{n} \\ & 0 \leq q_k \leq \bar{q} \\ & \text{for } k = 0, \dots, K-1 : \\ & n_{k+1} = F(y_n(kT), y_q(kT), u(kT), p) + w_k \end{aligned}$$

where  $k$  is the time interval counter of the MBPE,  $K$  is the identification horizon,  $w_k$ ,  $v_{n,k}$ , and  $v_{q,k}$  are vectors of auxiliary variables internal to the MBPE representing the process noise, and measurement noises associated with the accumulation state and inflow demand, respectively,  $Q$ ,  $R_n$ , and  $R_q$  are the inverse covariance matrices of the process noise, and measurement noises associated with the accumulation state and inflow demand, respectively,  $T$  is the sampling time,  $y_n(t)$  and  $y_q(t)$  are measurements on the accumulation state  $n(t)$  and inflow demand  $q(t)$ , respectively,  $n_k$  and  $q_k$  are the accumulation state and inflow demand vectors internal to the MBPE, respectively,  $\bar{n}$  and  $\bar{q}$  are upper bounds on the accumulation state and inflow demand (possibly obtained from an analysis on historical data), respectively,  $F$  is the discrete-time version of the dynamics given in (5), whereas  $u(t)$  is the known (recorded in the recent past) vector of perimeter control inputs.

Owing to the prediction error method involving one-step ahead predictions, and the dynamics (5) being linear in the MFD parameters, the MBPE problem (8) is a convex optimization problem that can be solved reliably and efficiently.

#### B. Moving Horizon Estimation

We formulate the problem of finding state estimate trajectories for a moving time horizon extending a fixed length into the past, striking a trade-off between measurements and the prediction model, as the following nonlinear MHE problem (based on the work in [38]):

$$\begin{aligned} \min_{w_k} \quad & \sum_{k=-N_e}^{-1} \|w_k\|_Q^2 + \sum_{k=-N_e}^0 \left( \|v_{n,k}\|_{R_n}^2 + \|v_{q,k}\|_{R_q}^2 \right) \quad (9) \\ \text{s.t.} \quad & \text{for } k = -N_e, \dots, 0 : \\ & y_n(t + kT) = n_k + v_{n,k} \\ & y_q(t + kT) = q_k + v_{q,k} \\ & 0 \leq n_k \leq \bar{n} \\ & 0 \leq q_k \leq \bar{q} \\ & \text{for } k = -N_e, \dots, -1 : \\ & n_{k+1} = F(n_k, q_k, u(t + kT), \hat{p}) + w_k \end{aligned}$$

where  $k$  is the time interval counter of the MHE,  $N_e$  is the estimation horizon,  $w_k$ ,  $v_{n,k}$ , and  $v_{q,k}$  are vectors of auxiliary variables internal to the MHE representing the process noise,

and measurement noises associated with the accumulation state and inflow demand, respectively,  $n_k$  and  $q_k$  are the accumulation state and inflow demand vectors internal to the MHE, respectively, whereas  $\hat{p}$  is the vector of model parameters obtained via MBPE as the solution of (8).

### C. Model Predictive Control

The problem of finding the control inputs that minimize total time spent (TTS) for a finite horizon can be formulated as the following economic nonlinear MPC problem (based on the work in [13] and [38]):

$$\begin{aligned}
 \min_{u_k} \quad & T \cdot \sum_{k=1}^{N_c} \mathbf{1}^T n_k \\
 \text{s.t.} \quad & n_0 = \hat{n}_t(t) \\
 & |u_0 - u(t - T)| \leq \Delta_u \\
 & \text{for } k = 0, \dots, N_c - 1 : \\
 & \quad n_{k+1} = F(n_k, \hat{q}_t(t), u_k, \hat{p}) \\
 & \quad \underline{u} \leq u_k \leq \bar{u} \\
 & \text{for } k = 1, \dots, N_c : \\
 & \quad n_{i,k} \leq n_{i,\text{jam}} \quad \forall i \in \mathcal{R},
 \end{aligned} \tag{10}$$

where  $k$  is the time interval counter of the MPC,  $N_c$  is the prediction horizon,  $n_k$  and  $u_k$  are the state and control input vectors internal to the MPC, respectively,  $\hat{n}_\tau(t)$  and  $\hat{q}_\tau(t)$  are estimates of the accumulation state  $n(t)$  and inflow demand  $q(t)$  for time  $\tau$  available at current time  $t$  (obtained via MHE as the solution of (9)),  $\Delta_u$  is the rate limiting parameter on control inputs,  $\underline{u}$  and  $\bar{u}$  are the control input constraints, whereas  $n_{i,k}$  is the total accumulation in region  $i$ .

Due to the nonlinear dynamics (5), the MHE and MPC problems given in eqs. (9) and (10), respectively, are nonconvex nonlinear optimization problems, which can be solved efficiently via, e.g., sequential quadratic programming or interior point solvers (for details, see [42]).

## IV. RESULTS

### A. Network Setup

An urban road network consisting of roughly 1500 links and 600 intersections is replicated as a computer model using the microscopic simulation package Aimsun. The model represents a portion of the urban network of the city of Barcelona in Spain, covering an area of 12 km<sup>2</sup>. The network is partitioned into four regions using the optimization-based clustering method of [23], where minimizing heterogeneity is considered in the objective function and cluster contiguity is enforced via constraints. The network model is taken from [39]; the reader is referred to that study for further details on the network.

### B. Identification Results

Accumulation state and inflow demand trajectories  $n(t)$  and  $q(t)$  are obtained by simulating a congested scenario for the network in using the Aimsun microscopic simulation framework. To reflect measurement noise, random noise terms  $v_n(t)$  and  $v_q(t)$  are added to the true trajectories to

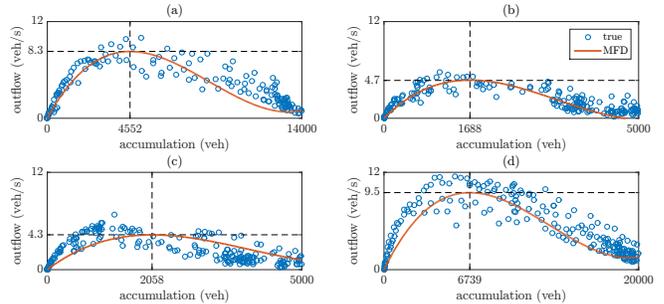


Fig. 1. Outflow as a function of regional accumulation for (a) region 1, (b) region 2, (c) region 3, (d) region 4, comparing the true outflow (blue), with the MFDs obtained by the MBPE method (red).

obtain the noisy measurements  $y_n(t)$  and  $y_q(t)$ . Covariances of the noise terms are chosen as  $\Sigma_{v_n} = I\sigma_{v_n}$  and  $\Sigma_{v_q} = I\sigma_{v_q}$ , with  $\sigma_{v_n} = 250$  veh and  $\sigma_{v_q} = 0.1$  veh/s, representing moderate amount of noise. A no control scenario is considered, which employs well-tuned fixed perimeter control input values. The network is empty at the beginning and faces increasing inflow demands. The demands are nonzero for the first 2 hours of the simulation, while the total simulation length is 8 hours to ensure that the network is empty also at the end of the simulation (to be able to compare different cases). With a sampling time of  $T = 90$  s, the simulation length is  $K = 320$  time steps. Given the measurements on accumulation and inflow demand trajectories, the optimization problem (8) is solved to obtain the MFD parameters.

The MFDs obtained by the proposed MBPE method are shown in fig. 1, where they are compared with the true values of the outflow as a function of regional accumulation  $g_i(n_i(t))$  (i.e., those obtained from microsimulation).

Overall, these results suggest that the proposed MBPE method is able to obtain MFDs that make physical sense, as they have a good qualitative match with the true outflows. However, the ultimate test for the obtained MFDs is usage in traffic estimation and control for improving mobility, which is examined in the following sections.

### C. Estimation and Control Results

The MFDs (i.e., the model parameters vector  $\hat{p}$ ) obtained by the MBPE method are used in the MHE and MPC schemes to control the urban network via perimeter control actuation for improving mobility under situations with noisy measurements. The MHE and MPC scheme is built using direct multiple shooting [43], while the dynamics are discretized with the Runge–Kutta method with a sampling time of  $T = 90$  s. The implementation is done using MPCTools [44], which is an interface to CasADi [45], with IPOPT [46] as solver, in MATLAB 8.5.0 (R2015a), on a 64-bit Windows PC with 3.6-GHz Intel Core i7 processor and 16-GB RAM. Estimation and prediction horizons are chosen as  $N_e = 20$  and  $N_c = 20$ , following the tuning results of [38] and [13], respectively. The perimeter controls are bounded via  $\underline{u} = 0.1$  and  $\bar{u} = 0.9$ , with a rate limit of  $\Delta_u = 0.1$ . Simulation length is  $K = 320$  in number of time steps,

corresponding to 8 hours of real time, as in the no control case. The combined estimation and control scenario is simulated using the Aimsun microscopic simulation framework together with stand-alone MHE and MPC executable generated using MATLAB. The simulation operates by evolving traffic conditions through Aimsun, where the MHE-MPC code is called every 90 seconds. The simulation operates by evolving traffic conditions through Aimsun, where the MHE-MPC code is called every 90 seconds. The MHE code, given recent measurements of accumulation  $y_n(t)$  and inflow demands  $y_q(t)$ , solves (9) to find the accumulation state and inflow demand estimates  $\hat{n}(t)$  and  $\hat{q}(t)$ . Then the MPC code, given  $\hat{n}(t)$  and  $\hat{q}(t)$ , solves (10) to find the perimeter control inputs  $u_k^*$  (with  $k = 0, \dots, N_c - 1$ ). Only the first one of these (i.e.,  $u_0^*$ ) is applied, which is realized in Aimsun by changing the duration of the green phases of the 28 predefined signalized intersections (out of about 600 in the network). The whole procedure is repeated at the next time step for 320 time steps.

The estimation results are shown in fig. 2, which depicts four selected accumulation trajectories (with measured, true, and estimated values) for a single Aimsun simulation with the combined MHE-MPC scheme. These figures suggest that the MHE scheme is able to obtain decent estimation performance, as indicated by the good match between the true and estimated accumulation state trajectories. Moreover, the regional accumulations for the no control, the combined MHE-MPC scheme, and the MPC with perfect measurement (named  $n$ -MPC) (i.e.,  $\hat{n}(t) = n(t)$  and  $\hat{q}(t) = q(t)$ ), are shown in fig. 3. The results indicate that MPC is capable of improving mobility compared to the no control case, as suggested by the regional accumulation trajectories, where it can decrease accumulations (and thus, the total time spent by vehicles in the network) and clear the network much faster. Overall, for the considered demand scenario, an improvement (in terms of total time spent in the network) of 25% is obtained through  $n$ -MPC and 20% for the MHE-MPC. These results indicate that: (a) The proposed MBPE method can lead to improved mobility when the MFD parameters obtained by the method are used in the MHE and MPC schemes, (b) the proposed MHE-MPC scheme is capable of traffic management under situations with noisy measurements, showing high potential for practical implementation. Moreover, the total CPU time of the MHE-MPC scheme is around 5 s (which is roughly negligible with respect to the sampling time of 90 s), indicating the real-time feasibility of the proposed scheme.

## V. CONCLUSION

In this paper we proposed application of an MBPE method to the identification of MFD-based dynamical models for large-scale urban road networks. Nonlinear MHE and MPC methods, employing the model parameters identified offline by the proposed MBPE method, are used in microscopic simulations for demonstrating applicability of the methods in a realistic setting. The results suggest potential for field applications of MFD-based feedback perimeter control.

Future work could include: (a) Comparisons of the proposed MBPE method with more standard approaches such as fitting polynomials for obtaining the MFDs, (b) sensitivity

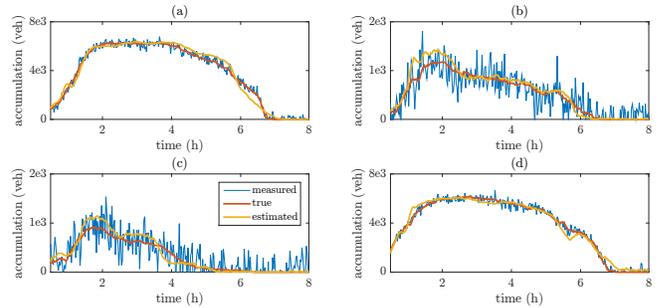


Fig. 2. Four selected accumulation state  $n_{ij}(t)$  trajectories of the Aimsun simulation with combined MHE-MPC scheme, showing the measured (blue), true (red), and estimated (yellow) values of: (a)  $n_{14}(t)$ , (b)  $n_{23}(t)$ , (c)  $n_{32}(t)$ , (d)  $n_{44}(t)$ .

analyses of MFD parameter estimation methods to noisy measurements, (c) testing various MFD-based models in model-based control of congested networks.

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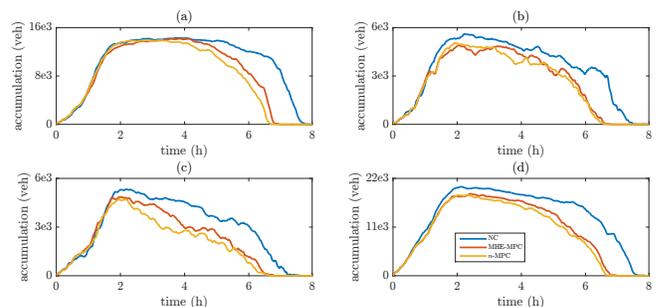


Fig. 3. Regional accumulation state  $n_i(t)$  trajectories of the Aimsun simulations with the no control (NC) case (blue), combined MHE-MPC scheme (red), and MPC with perfect measurement ( $n$ -MPC, yellow), showing: (a)  $n_1(t)$ , (b)  $n_2(t)$ , (c)  $n_3(t)$ , (d)  $n_4(t)$ .

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