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EEM/EEE314 Automatic Control Systems

Exam-style questions with solutions

Part 3: Electromechanical systems

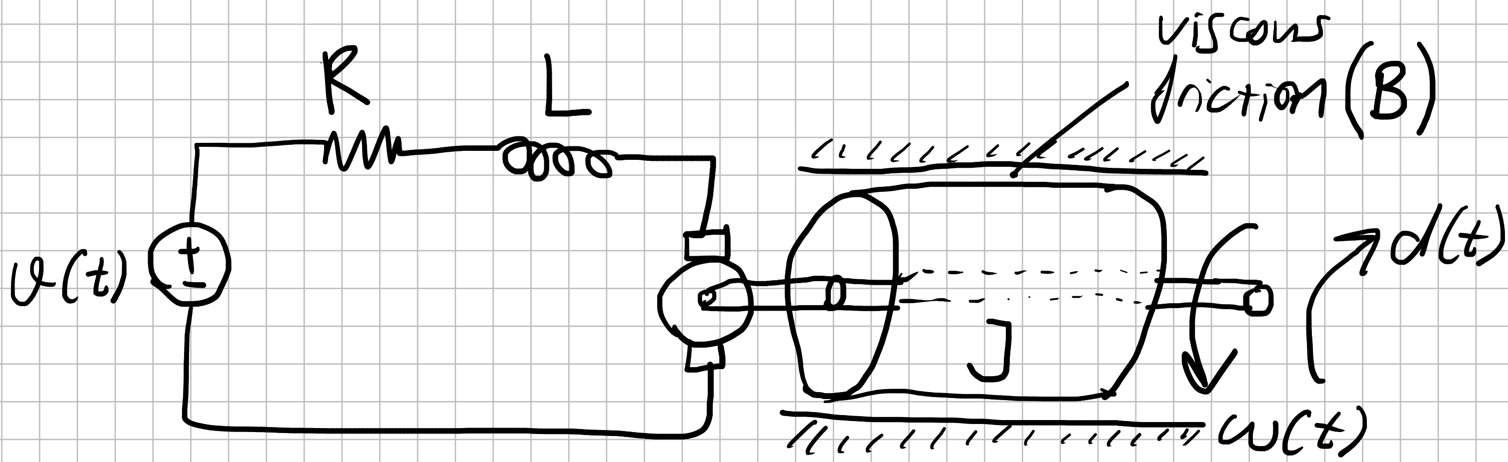
Abbreviations:

emf: electromotive force

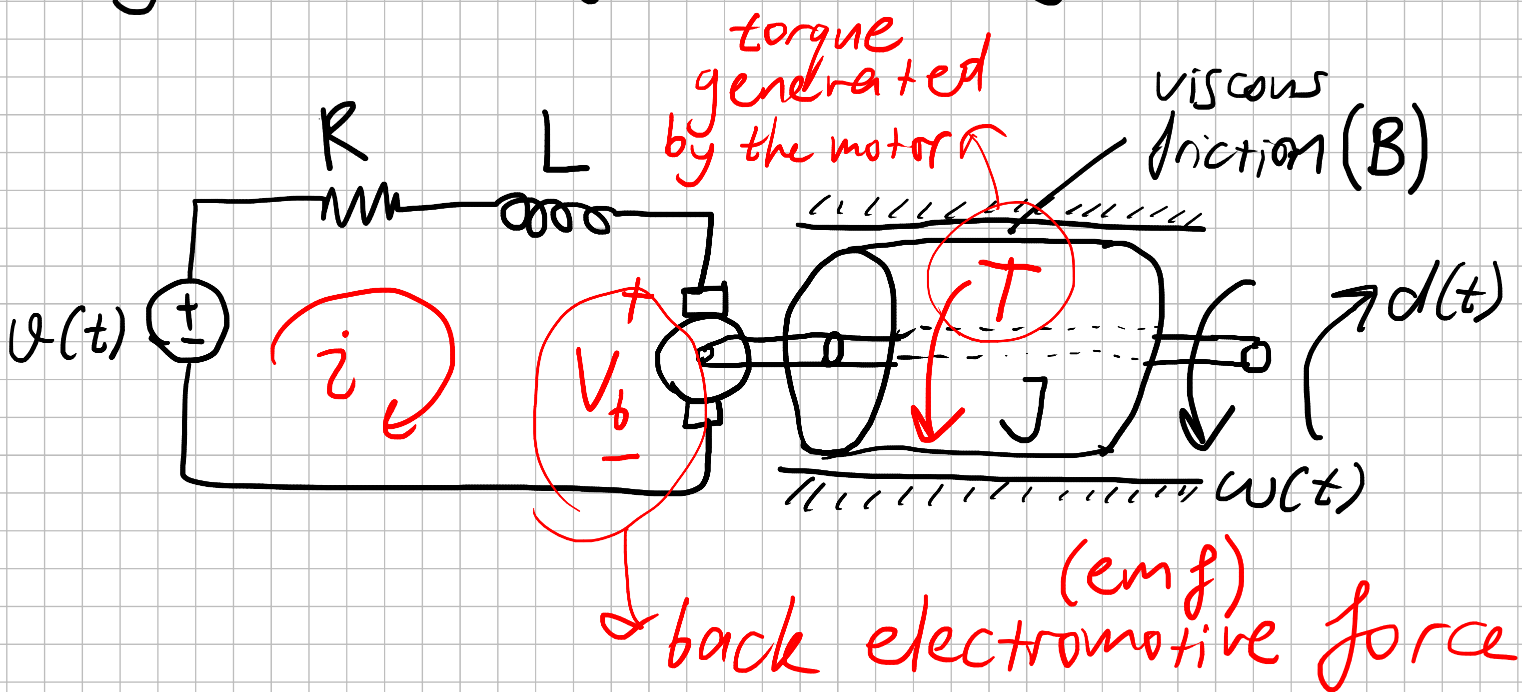
KVL: Kirchhoff's voltage law

N2L: Newton's second law of motion

Question 1: Consider the schematic of an rotational electromechanical system (DC motor) depicted below, consisting of a resistor and an inductor (with parameters R and L) for the electrical part, and a rotating object with a moment of inertia of J and a viscous friction element with constant B, for the mechanical part. The motor torque constant Km is equal to the back electromotive force constant Kb, and is given as K (that is, K = Km = Kb). Angular velocity of the mass is denoted as ω(t).
An external voltage is being applied to the system, denoted as v(t). An external load torque is being applied to the system, denoted as d(t). There are no other forces or torques acting on the system.
Find the differential equation model of the system relating v(t), d(t) and ω(t).



Solution: Define the following symbols for the system:



Note that T and ω have the same direction, while d is in the opposite direction.

↳ We should notice this from the system schematic given in the question.

Writing KVL for the electrical part:

$$v = V_R + V_L + V_b$$

From linear models of R and L elements:

$$V_R = R \cdot i \qquad V_L = L \cdot \dot{i}$$

Thus, KVL becomes:

$$v = R \cdot i + L \cdot \dot{i} + V_b$$

Writing N2L for the rotational motion of the mechanical part:

$$\sum_i \tau = J \cdot \dot{\omega}$$

$$\underbrace{T}_{\substack{\text{torque generated}\\ \text{by the motor}}} - \underbrace{d}_{\substack{\text{external load}\\ \text{torque}}} - \underbrace{B \cdot \omega}_{\substack{\text{torque due to}\\ \text{viscous friction}}} = J \cdot \dot{\omega}$$

The torque-current relationship is as follows: $T = K \cdot i$

The back emf-speed relationship is as follows: $V_b = K \cdot \omega$

Substituting these into KVL and N2L:

$$v = R \cdot i + L \cdot \dot{i} + K \cdot \omega$$

$$J\dot{\omega} + B\omega - K \cdot i = -d$$

Rewriting this for i alone:

$$i = \frac{J\dot{\omega} + B\omega + d}{K}$$

Differentiating both sides:

$$\dot{i} = \frac{J\ddot{\omega} + B\dot{\omega} + \dot{d}}{K}$$

Substituting these in KVL:

$$v = \frac{RJ}{K}\dot{\omega} + \frac{RB}{K}\omega + \frac{R}{K}d + \ldots$$

$$+ \frac{LJ}{K}\ddot{\omega} + \frac{LB}{K}\dot{\omega} + \frac{L}{K}\dot{d} + K\omega$$

Simplifying by gathering ω and $\dot{\omega}$ terms together:

$$\boxed{\begin{gathered} \frac{LJ}{K}\ddot{\omega} + \frac{RJ+LB}{K}\dot{\omega} + \left(\frac{RB}{K} + K\right)\omega = \ldots \\ \ldots v - \frac{L}{K}\dot{d} - \frac{R}{K}d \end{gathered}}$$