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EEM/EEE314 Automatic Control Systems

Exam-style questions with solutions

Part 2: Mechanical systems

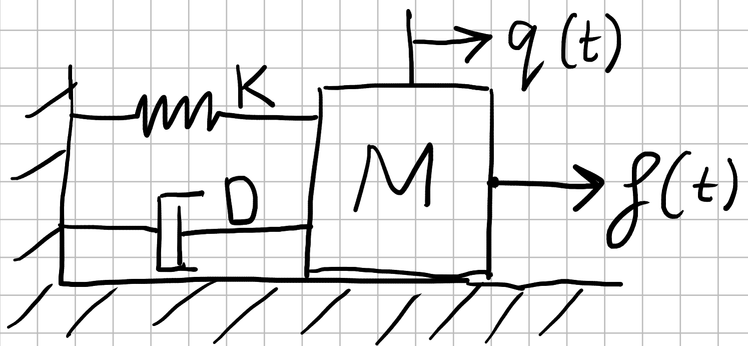
Abbreviations:

FBD: free body diagram

N2L: Newton's second law of motion

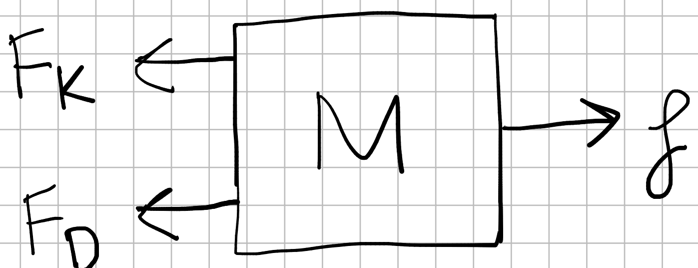
LHS: left-hand side (of the equation)

Question 1: Consider the schematic of a translational mechanical system depicted below, consisting of a mass, a spring, and a damper (with parameters M, K, and D, respectively). Position of the mass is denoted as q(t). An external force is being applied to the system, denoted as f(t). There are no other forces acting on the system. Find the differential equation model of the system relating f(t) and q(t).



Solution:

Draw the FBD for the mass:



From linear models of spring & damper, we can write:

$$F_K = K \cdot q$$

$$F_D = D \cdot \dot{q}$$

Writing N2L for the mass:

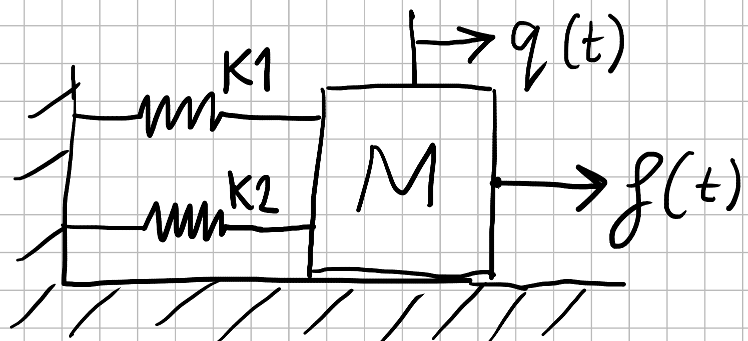
$$\sum_i F_i = M a = M \ddot{q}$$

$$f - K \cdot q - D \cdot \dot{q} = M \ddot{q}$$

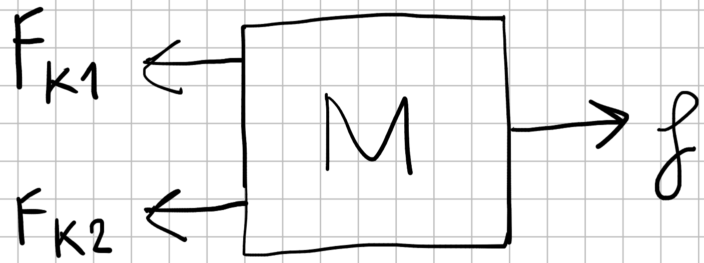
or (rearranging to have q terms on LHS):

$$\boxed{M \cdot \ddot{q} + D \cdot \dot{q} + K q = f}$$

Question 2: Consider the schematic of a translational mechanical system depicted below, consisting of a mass and two springs (with parameters M, K1, and K2, respectively). Position of the mass is denoted as q(t). An external force is being applied to the system, denoted as f(t). There are no other forces acting on the system.
Find the differential equation model of the system relating f(t) and q(t).



Solution: Draw FBD of the mass



From linear model of spring we can write:

$$F_{K1} = K1 \cdot q \qquad F_{K2} = K2 \cdot q$$

Writing N2L for the mass:

$$\sum_i F_i = Ma = M\ddot{q}$$

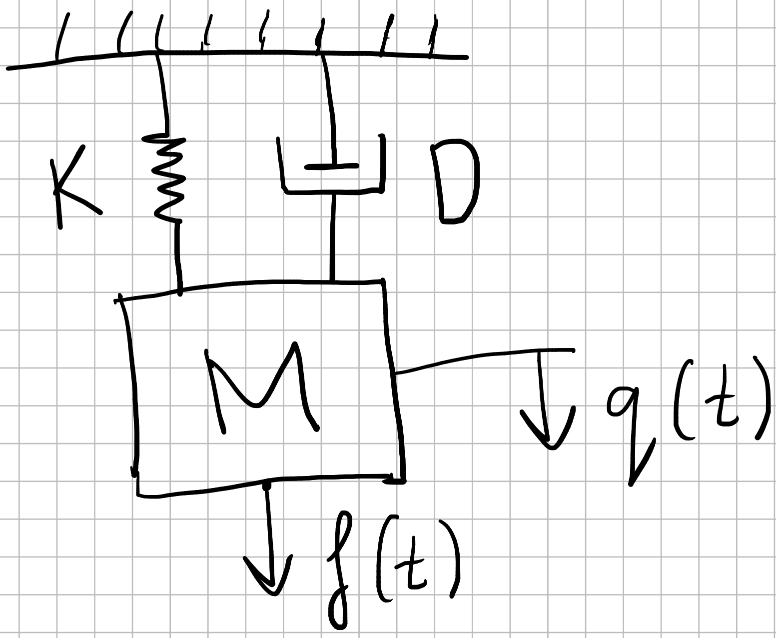
$$f - K1 \cdot q - K2 \cdot q = M \cdot \ddot{q}$$

$$f - (K1 + K2) \cdot q = M \cdot \ddot{q}$$

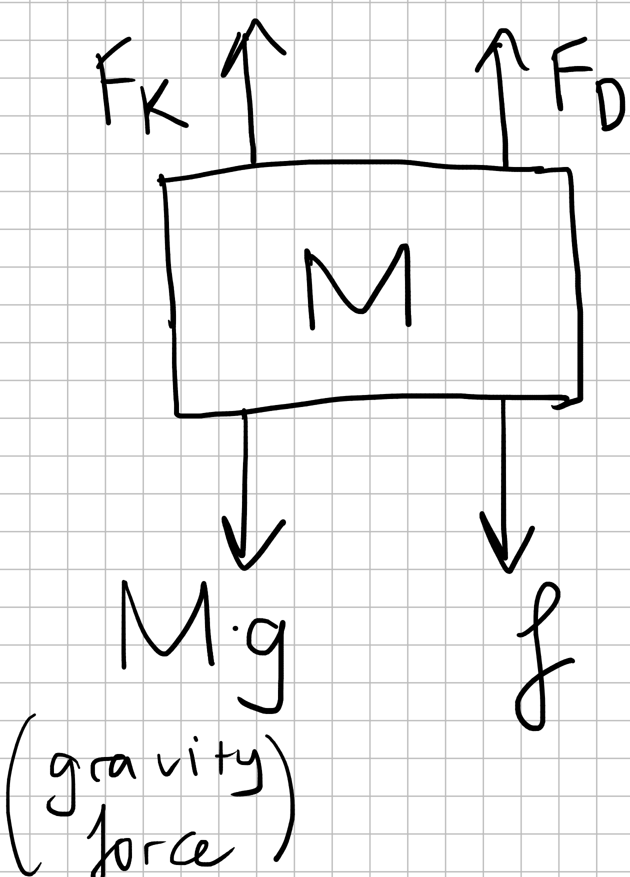
or (rearranging to have q terms on LHS):

$$\boxed{M \cdot \ddot{q} + (K1 + K2) \cdot q = f}$$

Question 3: Consider the schematic of a translational mechanical system depicted below, consisting of a mass, a spring, and a damper (with parameters M, K, and D, respectively). Position of the mass is denoted as q(t). An external force is being applied to the system, denoted as f(t). Gravity is acting on the system, with gravitational acceleration constant g.
There are no other forces acting on the system.
Find the differential equation model of the system relating f(t) and q(t).



Solution: Draw FBD for the mass



From linear models of spring & damper, we can write:

$$F_K = K \cdot q$$

$$F_D = D \cdot \dot{q}$$

Writing N2L for the mass:

$$\sum_i F_i = Ma = M\ddot{q}$$

$$f + Mg - K \cdot q - D \cdot \dot{q} = M\ddot{q}$$

or (rearranging to have q terms on LHS):

$$\boxed{M \cdot \ddot{q} + D \cdot \dot{q} + Kq = f + Mg}$$